Lecture Notes for Chapter 2

International Financial Markets and Institutions

Chapter 2

Preliminaries: Conventions, notation, and basic concepts

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Road Map

12 Summary of international finance

13 Revision classes

2.1 Main Issues

- Notation for referring to currencies
- Convention for quoting exchange rates
- Symbols and notation for random variables
  - This notation will allow us to be precise and minimize confusion
- Discounting riskless cashflows.
- Review of expected values, standard deviation, variance, correlation and covariance
  - Standard deviation as a measure of risk
  - Review of CAPM, systematic and unsystematic risk

  - Measuring systematic risk of a stock's return via its beta

- Discounting risky cashflows.

- Arbitrage

  - a concept that we will use throughout the text, and

- Regression analysis

  - a tool that we will use often

2.2 Currencies

One of the main objects of interest will be the currency of a particular country. Given that we are in an international setting, we will wish to discuss the currencies of two countries. To keep our discussion entirely general, we will label the currencies of the two countries as:

- HC: the currency of the $h$ome $c$ountry
- FC: the currency of the $f$oreign $c$ountry

- Advantage: do not have to assume that the analysis applies only to a particular country.

  If you live in the United States, then $\textsc{hc}$ = USD, the US Dollar.
  If you live in Europe, the $\textsc{hc}$ = EUR, the Euro.
  For those of us fortunate enough to live in Canada, $\textsc{hc}$ = CAD, the Canadian Dollar.

  In the same way, denoting the foreign currency as $\textsc{fc}$ means that we can apply our analysis to any foreign country.

- Of course, when we look at *real world data* we will have to consider *particular exchange rates*. In that case, we will use the following notation for the exchange rates of different countries

Notation	Currency
AUD	Australian Dollar
BEF	Belgian Franc
CAD	Canadian Dollar
DEM	German Mark
EUR	Euro for the Eurozone
FRF	French Franc
GBP	British Pound
JPY	Japanese Yen
NZD	New Zealand Dollar
USD	US Dollar

The currencies: BEF, DEM, FRF are no longer traded, but are important historically.

2.2.1 The exchange rate

- An important quantity will be the exchange rate—the rate at which one can exchange one currency for another.

- Our convention: quote exchange rates in terms of HC/FC.
  - This quote tells us the price of the FC.
  - Thus, statements about buying or selling will always refer to the currency in the denominator (the "foreign" currency).
  - This convention, standard in continental Europe, is called the *direct quote*.

▶ To keep things clear, you should remember that the object of interest (the currency that we are buying or selling) is the one that is in the denominator.

Example 2.1 (Exchange rate)

– Just as

  ⋆ USD/car—is the price for a car with the "item" being bought or sold, in this case a car, in the denominator,
  ⋆ GBP/share—is the price (in Pounds) for one one share, with the share in the denominator;

– For exchange rates we have:

  ⋆ USD/GBP is the price for one British Pound,
  ⋆ JPY/CAD is the price in Yen for one Canadian Dollar, and
  ⋆ JPY/EUR is the price in Yen for one Euro.

2.3 Symbols used

Usually, single characters from the roman script will be used to refer to quantities of interest.

Symbol	Refers to
S	spot rate to exchange 1 unit of FC for S units of HC
F	forward rate
f	price of a futures contract
c	price of a call option
p	price of a put option
r	the riskless interest rate
C	a generic cashflow

Observe that we distinguish between lower-case and upper-case characters, so that F denotes a forward exchange rate whereas f denotes the price of a futures contract.

Also, to distinguish between quantities for the home country and those for the foreign country, we will use the superscript * to label variables for the foreign country.

Example 2.2 (Notation—variables for foreign country)
The interest rate for the foreign country is denoted by r^*, while that for the home country is r.

If the home country is Japan, then a cashflow denominated in JPY would be denoted C while that in a different country as C^*.

2.3.1 Subscripts

Symbols with a time-dimension will be subscripted. The subscripts used are:

Subscript	Refers to
t	today
$t+1$	one period from today (tomorrow, next month, or next year)
$t-1$	one period before today (yesterday, last month, or last year)
T	the *t*erminal date or the maturity date

For convenience, we will often denote the date today as $t = 0$.

We can now see how it is possible to combine the two sets of notation above:

Symbol	Refers to
S_t	spot rate today
S_0	spot rate at time 0 (today)
S_{t+1}	spot rate next period
$F_{t,t+1}$	forward rate today, for a contract maturing next period
$f_{t,t+1}$	price today for a futures contract maturing at $t+1$
c_t	price today of a call option
p_t	price today of a put option
$r_{t,t+1}$	the riskless interest rate over the time interval t to $t+1$
C_0	a cashflow at $t = 0$ (today)
C_{t+3}	a cashflow coming three periods from today

2.4 Review of NPV analysis I

- We review NPV analysis from COMM 397, i.e. *ignoring* international issues such as the exchange rate.
- Whilst we do this review think about how international issues could change the way we do NPV analysis.
- The key concepts we refer to when discounting cashflows are *time* and *risk*.
  - When discounting a cashflow to compute its present value, the discount factor you use will depend on when the cashflow occurs (time) and how risky it is.

2.4.1 Computing the cashflows

Remember to use

- After-tax cashflows.
- Use cashflows, not accounting earnings.
- Use cashflows attributable to the project.
  - Use incremental cashflows.
  - Ignore sunk costs.
  - Include investment in working capital as expenditure.
  - Include opportunity costs of using existing equipment.

2.4.2 Discounting riskless cashflows

Suppose you (a Canadian resident) will definitely receive CAD 1000 in 2 years from now.

What discount factor would you use to compute the present value of this riskless cashflow?

You could look up the yield-to-maturity on a 2 year Canadian Goverment Bond issued now and which expires in 2 years.

Suppose the YTM is 3.00%.

The discount factor you would use is then

$\frac{1}{(1+3.00\%)^2}$

Hence, the PV of the riskless cashflow is

- $\textsc{cad}1000 \times \frac{1}{(1+3.00\%)^2} = \textsc{cad}942.60$

2.4.3 Discounting risky cashflows

When discounting risky cashflows, in addition to the time value of money, we must consider the riskiness of the cashflows.

The main question we must answer is: how does the riskiness of cashflows affect the discount factor?

In order to answer this question, we study random variables, so we can understand the concept of risk.

2.5 Random variables

We will want to distinguish between riskless quantities (that are known today) and risky quantities (whose value is not known today). To indicate that a particular quantity is uncertain today, we will use $\tilde{\cdot}$ over it.

Symbol	Refers to
S_t	spot rate today—*not* risky
$\tilde{S}_{t+1}$	spot rate next period that is not known today
$F_{t,t+1}$	forward rate quoted today (and therefore it is known), for a contract maturing next period
$\tilde{F}_{t+1,t+2}$	forward rate to be quoted tomorrow (and so not known), for a contract maturing the following period
$\tilde{f}_{t+1,T}$	price that will prevail tomorrow for a contract maturing at T
$\tilde{r}_{t+1,t+2}$	the future riskless one period interest rate that will prevail tomorrow
$\tilde{C}_{t+2}$	a future cashflow two periods from now

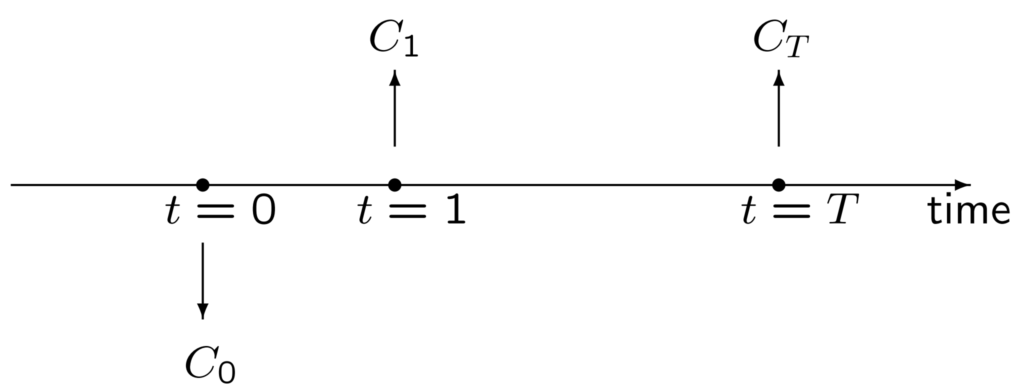
We shall review how to compute the following:

- The expected value of a random variable.
- The certainty equivalent of a random variable.
- The standard deviation and variance of random variable.
- The covariance and correlation between two random variables.

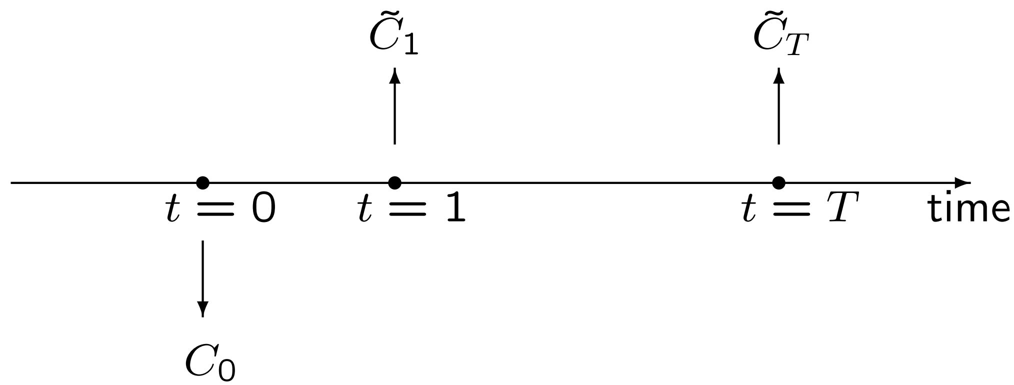
2.5.1 Arrow notation for visualizing cash flows at different points in time

We will use a time line with arrows to indicate when a particular *inflow (up-arrow)* or *outflow (down-arrow)* occurs.

In the figure below, there is a (known) outflow of C_0 at time zero, followed by two (known) inflows at $t = 1$ and at T.



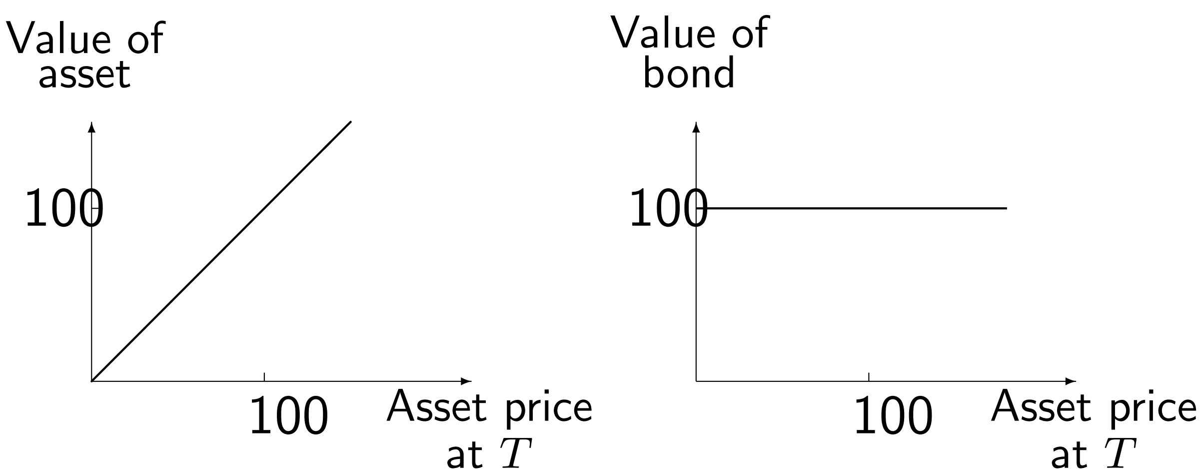
In this figure, there is a (known) outflow of C_0 at time zero, followed by two uncertain inflows at $t = 1$ and at T.



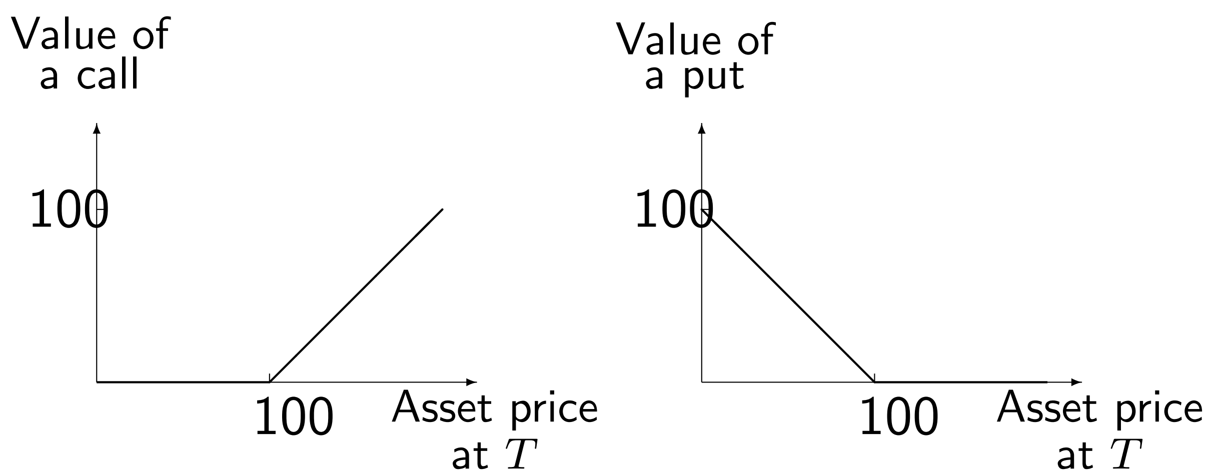
2.5.2 Payoff diagrams for visualizing cash flows across states

Another kind of picture that is useful in understanding payoffs is one where we plot the value of the position as a function of some other price. This is called a *payoff diagram* or *profile*.

- A stock and a bond (with face value \$100) have the following payoff diagrams.



- Payoffs of calls and puts (with strike price 100) can be described by plotting their payoffs at expiration as a function of the price of the underlying asset.



▶ Observe that these pictures show the value at only a *single* point in time.

2.5.3 Expected value of a random variable

When the value of a future quantity is uncertain, one can form expectations today about this quantity. We will denote by $\mathrm{E}_t\tilde{X}_{t+1}$ the expectation today of a future quantity $\tilde{X}_{t+1}$ that is uncertain.

Example 2.3 (Notation—expectation)
Here are some more examples of the use of notation involving the expectations operator.

Symbol	Refers to
$\mathrm{E}_t\widetilde{S}_{t+1}$	expectation today of next period's spot rate
$\mathrm{E}_t\widetilde{F}_{t+1,t+2}$	expectation today of the next period's forward rate
	for a contract maturing the following period
$\mathrm{E}_t\tilde{r}_{t+1,t+2}$	expectation today of the future riskless one period interest rate
$\mathrm{E}_t\widetilde{C}_{t+2}$	expectation today of a future cashflow two periods from now

Example 2.4 (Computing expectations)

Consider a throwing a fair coin. The probability of getting heads is $1/2$ and the probability of getting tails is $1/2$. If you were to obtain HC 200 if the coin lands on heads and zero otherwise, what is the expected value of this 'gamble'?

The expected value of the gamble is

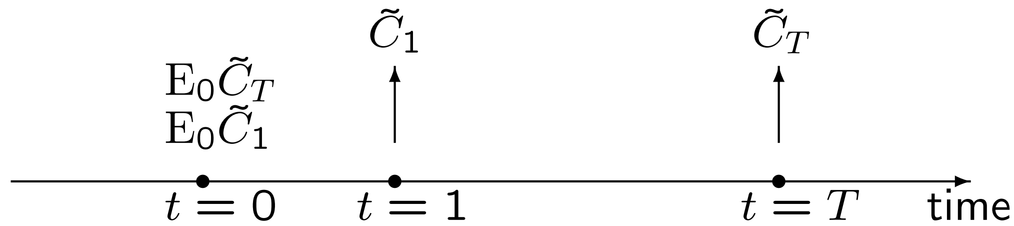
- $1/2 \times \text{HC}200 + 1/2 \times 0 = \text{HC}100$

  Now suppose the coin is unfair and the probability of getting heads is $1/4$. If you were to obtain HC 200 if the coin lands on heads and zero otherwise, what is the expected value of this 'gamble'?

The expected value of the gamble is

- $1/4 \times \textsc{hc}200 + 3/4 \times 0 = \textsc{hc}50$

We can also show expectations on a time line: suppose there are two uncertain inflows at $t = 1$ and at T, whose expectation at time 0 is $\mathrm{E}_0\tilde{C}_1$ and $\mathrm{E}_0\tilde{C}_T$.



2.5.4 Certainty equivalent value of a random variable

The expectations above do not adjust for risk, but we know that most investors are *risk averse*.

Example 2.5 (Risk aversion)
Consider the following two choices:

- Choice A: HC 100 to be received with certainty

- Choice B: Toss a coin, and if it is heads receive HC 200 but if it is tails receive HC 0.

Then risk averse agents will prefer Choice A since it has the same expected value as Choice B, but it has less (zero) risk.

Definition 2.1 (Certainty equivalent)
The certain amount that a risk averse agent would consider *equivalent* to a risky payoff is called the certainty equivalent.

Example 2.6 (Certainty equivalent)
Consider the following two choices:

– Choice B: Toss a coin, and if it is heads receive HC 200 but if it is tails receive HC 0.

– Choice C: HC x to be received with certainty.

Then what does x need to be so that an agent is indifferent between Choice B and Choice C?

Suppose that if $x = \text{HC}$ 90, then the agent is indifferent between Choices B and C. That is, if you asked the agent what she would be willing to give you in exchange for Choice B, she would offer you HC 90.

Then, we would say that the certainty equivalent (CEQ) value of Choice B is HC 90:

$$\mathrm{CEQ}(\text{Choice B}) = \text{HC } 90.$$

- When the value of a future quantity is uncertain, one can form *risk-adjusted* expectations today about this quantity. We will denote by $\mathrm{CEQ}_t(\tilde{X}_{t+1})$ the risk-adjusted expectation today of a future quantity $\tilde{X}_{t+1}$ that is uncertain.

Example 2.7 (Notation—certainty equivalent)

Here are some more examples of the use of notation involving the risk-adjusted expectations operator.

Symbol	Refers to
$\mathrm{CEQ}_t(\tilde{S}_{t+1})$	risk-adjusted expectation today of next period's spot rate
$\mathrm{CEQ}_t(\tilde{F}_{t+1,t+2})$	risk-adjusted expectation today of next period's forward rate
	for a contract maturing the following period
$\mathrm{CEQ}_t(\tilde{r}_{t+1,t+2})$	risk-adjusted expectation today
	of the future riskless one period interest rate

Risk-adjusted expectations can also be shown on a time line. Suppose there is an uncertain inflow at T, whose certainty equivalent value at time 0 is $\mathrm{CEQ}_0(\tilde{C}_T)$.


$\tilde{C}_T$
$\mathrm{CEQ}_0(\tilde{C}_T)$
$t = 0$
$t = T$
time

Example 2.8 (Numerical example—certainty equivalent)
An investor obtains 'utility' $u(x) = 2\sqrt{x}$ from HC x.

The investor tosses a fair coin, and if it is heads he receives HC 200 but if it is tails he receives HC 0.

What is the certainty equivalent of this gamble?

First compute the expected utility of this gamble:

- $1/2 \times u(200) + 1/2 \times u(0) = 1/2 \times 2 \times \sqrt{200} + 1/2 \times 0 = \sqrt{200} = 14.14$

The certainty equivalent is the certain amount HC x such that the investor has utility $u(x) = \sqrt{200}$ i.e. $2\sqrt{x} = \sqrt{200}$.

Hence, $x = (1/2 \times \sqrt{200})^2 = 1/4 \times 200 = 50$.

2.5.5 Standard deviation and variance of a random variable

Variance is a measure of how much a random variable deviates from its expected value. Standard deviation is the square root of variance.

$Var_t\tilde{X}_{t+1}$ denotes the variance today of a future quantity $\tilde{X}_{t+1}$ that is uncertain.

Variance is defined as the expected value of squared deviations from the mean.

- $$\boxed{Var_t\tilde{X}_{t+1} = \mathrm{E}_t(\tilde{X}_{t+1} - \mathrm{E}_t\tilde{X}_{t+1})^2}$$

  We will denote by $\sigma_t\tilde{X}_{t+1}$ the standard deviation today of a future quantity $\tilde{X}_{t+1}$ that is uncertain.

- $$\boxed{\sigma_t\tilde{X}_{t+1} = \sqrt{Var_t\tilde{X}_{t+1}}}$$

In finance, standard deviation and variance are used as measures of 'risk'.

Example 2.9 (Numerical example–standard deviation and variance)
Consider once again, throwing a fair coin. The probability of getting heads is $1/2$ and the probability of getting tails is $1/2$. If you were to obtain HC 200 if the coin lands on heads and zero otherwise, what is the standard deviation of this 'gamble'?

We know that the expected value is HC 100.

Denote by $\tilde{X}$, the outcome of the gamble. Hence

$$\tilde{X} = \begin{pmatrix} \text{HC} & 200 & \textit{with} & \textit{probability} & 1/2 \\ \text{HC} & 0 & \textit{with} & \textit{probability} & 1/2 \end{pmatrix}$$

Therefore

$$\tilde{X} - \mathrm{E}\tilde{X} = \begin{pmatrix} \text{HC} & 100 & \textit{with} & \textit{probability} & 1/2 \\ \text{HC} & -100 & \textit{with} & \textit{probability} & 1/2 \end{pmatrix}$$

We can now compute $Var\tilde{X} = \mathrm{E}(\tilde{X} - \mathrm{E}\tilde{X})^2$.

$$Var\tilde{X} = 1/2(\text{HC}100)^2 + 1/2(-\text{HC}100)^2 = (\text{HC}100)^2$$

Hence, the standard deviation is given by

$$\sigma(\tilde{X}) = \text{HC}100.$$

Note that variance has the units of expected value squared. By defining standard deviation as the square root of variance, we obtain a measure of deviation from the expected value, which has the same units as expected value.

2.5.6 Covariance and correlation between two random variables

Consider a risky stock with return given by the random variable $\tilde{R}$. Let $\tilde{R}_M$ denote the return on the stock market.

The table below gives the return for both the individual stock and the market in each state of the world.

State	1	2	3
Probability	0.20	0.60	0.20
$\tilde{R}$	- 5.00 %	10.00%	20.00%
$\tilde{R}_M$	-10.00%	10.00%	40.00%

The two random variables appear to move together. We would like to measure to what extent they *covary*. The *covariance* is an attempt to quantify this concept – it tells us *how two random variables behave in relation to each other*.

The covariance between $\tilde{R}$ and $\tilde{R}_M$ is defined as the expectation of the product of the deviations about the mean of the two random variables.

- $$\boxed{Cov(\tilde{R}, \tilde{R}_M) = \mathrm{E}[(\tilde{R} - \mathrm{E}(\tilde{R}))(\tilde{R}_M - \mathrm{E}(\tilde{R}_M))]}$$

Using the definition of covariance, compute the covariance between returns on the individual stock and the market.

First we compute expectations:

- $\mathrm{E}\tilde{R} = 0.20 \times (-5.00\%) + 0.60 \times 10.00\% + 0.20 \times 20.00\% = -1.00\% + 6.00\% + 4.00\% = 9.00\%$

  and

- $\mathrm{E}\tilde{R_M} = 0.20 \times (-10.00\%) + 0.60 \times 10.00\% + 0.20 \times 40.00\% = -2.00\% + 6.00\% + 8.00\% = 12.00\%$

We can now write down the deviations from the expected value for each random variable, state–by–state.

State	1	2	3
Probability	0.20	0.60	0.20
$\tilde{R} - \mathrm{E}\tilde{R}$	- 14.00%	1.00%	11.00%
$\tilde{R_M} - \mathrm{E}\tilde{R}_M$	-22.00%	-2.00%	28.00%

Hence, we can calculate the covariance:

- $Cov(\tilde{R}, \tilde{R}_M) = (0.20 \times (-14.00) \times (-22.00) + 0.60 \times 1.00 \times (-2.00) + 0.20 \times 11.00 \times 28.00)\%^2 = 122 \times \%^2$

The covariance is hard to interpret and it does not have the same units as returns, so we define a quantity called correlation.

The *correlation* between 2 random variables is a unit-free measure of the strength and direction of the linear relationship between them.

Correlation lies between -1 for a perfectly negative relationship, through zero, where the 2 variables are independent of each other, to $+1$ for a perfectly positive relationship between the variables.

The correlation coefficient ρ_{R,R_M}, is calculated by dividing the covariance between $\tilde{R}$ and $\tilde{R}_M$ by the product of the standard deviation of $\tilde{R}$ and the standard deviation of $\tilde{R}_M$, i.e.

- $\boxed{\rho_{R,R_M} = \frac{Cov(\tilde{R},\tilde{R}_M)}{\sigma(\tilde{R})\sigma(\tilde{R}_M)}}$

Calculate the correlation between returns on the individual stock and the market.

We have already calculated the covariance, so all that remains is calculate the standard deviations.

- $Var(\tilde{R}) = \mathrm{E}(\tilde{R} - \mathrm{E}\tilde{R})^2 = (0.20 \times (-14.00)^2 + 0.60 \times (1.00)^2 + 0.20 \times (11.00)^2) \times \%^2 = 64\%^2$
- $Var(\tilde{R}_M) = \mathrm{E}(\tilde{R}_M - \mathrm{E}\tilde{R}_M)^2 = (0.20 \times (-22.00)^2 + 0.60 \times (-2.00)^2 + 0.20 \times (28.00)^2) \times \%^2 = 256\%^2$

Hence,

- $\sigma(\tilde{R}) = 8.00\%$
- $\sigma(\tilde{R}_M) = 16.00\%$
- $\rho_{R,R_M} = \frac{Cov(\tilde{R},\tilde{R}_M)}{\sigma(\tilde{R})\sigma(\tilde{R}_M)} = \frac{122.00}{8.00 \times 16.00} = .9531$

Example 2.10 (Capital asset pricing model)

How do we measure the performance of a risky asset such as a stock? We look at the returns on this stock and compare them to the riskless rate (returns on riskless security, such as Government bonds).

The return on a stock relative to the riskfree rate is called the *excess return* or *risk premium*.

Denote by $\tilde{R}_{i,t+1}$ the risky return at date $t+1$ on stock i.

$\tilde{R}_{i,t+1} = \frac{\tilde{P}_{i,t+1}+\tilde{D}_{i,t+1}-P_{i,t}}{P_{i,t}}$,

where $P_{i,t}$ is the date t price of stock i and $D_{i,t}$ is the date t dividend payout.

r_t is the riskless rate at date t.

Hence $R_{i,t} - r_t$ is the date t excess return or risk premium of stock i.

- The *riskiness* of a stock is given by the *standard deviation* of its returns.

The higher the standard deviation of its returns, the more risky the stock.

The concept of risk is central to finance. This is why we have looked at expected values and standard deviations.

The capital asset pricing model relates the risk premium on an individual stock to the risk premium of the stock market.

The CAPM gives a relationship between *individual* stock performance and *aggregate* stock market performance.

Denote the date t return on the stock market by $R_{M,t}$

► $$\mathrm{E}_t(\tilde{R}_{i,t+1}) - r_{t+1} = \beta_{i,t}(\mathrm{E}_t(\tilde{R}_{M,t+1}) - r_{t+1}),$$
where
$$\beta_{i,t} = \frac{\mathrm{Cov}_t(\tilde{R}_{i,t+1}, \tilde{R}_{M,t+1})}{\mathrm{Var}_t(\tilde{R}_{M,t+1})}.$$

The expected risk premium on a stock is related to the expected market risk premium by the stock's beta.

To understand what beta measures, recall that the riskiness of future stock returns, $\tilde{R}_{i,t+1}$, is given by the standard deviation, $\sigma_t(\tilde{R}_{i,t+1})$.

The unexpected portion of the stock's return contains two components: the first coming from systematic risk, $\tilde{M}_{i,t+1}$, and the second from unsystematic risk, $\tilde{\epsilon}_{i,t+1}$.

$$\tilde{R}_{i,t+1} = \mathrm{E}_t\tilde{R}_{i,t+1} + \tilde{M}_{i,t+1} + \tilde{\epsilon}_{i,t+1}$$

Unsystematic risk is essentially eliminated by diversification, so only systematic risk is priced.

The beta of the stock reflects this–we can show that the beta of a stock is just its systematic risk, i.e.

$$\beta_{i,t} = \frac{Cov_t(\tilde{R}_{i,t+1}, \tilde{R}_{M,t+1})}{Var_t(\tilde{R}_{M,t+1})} = \sigma_{t+1}(\tilde{M}_{i,t+1}).$$

Proof (non-examinable)

If only systematic risk is priced, then

$Cov_t(\tilde{\epsilon}_{i,t+1}, \tilde{R}_{M,t+1}) = 0.$

Hence,

$$Cov_t(\tilde{R}_{i,t+1}, \tilde{R}_{M,t+1}) = Cov_t(\tilde{M}_{i,t+1}, \tilde{R}_{M,t+1}) = \rho_{M_i,R_M}\sigma_{t+1}(\tilde{M}_{i,t+1})\sigma_{t+1}(\tilde{R}_{M,t+1})$$

The systematic risk of the stock is perfectly correlated with market returns, so

$\rho_{M_i,R_M} = 1$

Hence

$$Cov_t(\tilde{R}_{i,t+1}, \tilde{R}_{M,t+1}) = \sigma_{t+1}(\tilde{M}_{i,t+1})\sigma_{t+1}(\tilde{R}_{M,t+1}).$$

This means that the beta of the stock is given by

$$\frac{\sigma_{t+1}(\tilde{M}_{i,t+1})\sigma_{t+1}(\tilde{R}_{M,t+1})}{\sigma_{t+1}(\tilde{R}_{M,t+1})} = \sigma_{t+1}(\tilde{M}_{i,t+1}),$$

which is just the systematic risk of the stock. $\square$

Using the table below, which gives the return for both the individual stock and the market in each state of the world, calculate the stock's beta.

State	1	2	3
Probability	0.20	0.60	0.20
$\tilde{R}_{i,t+1}$	- 5.00 %	10.00%	20.00%
$\tilde{R}_{M,t+1}$	-10.00%	10.00%	40.00%

We know that

$$Cov_t(\tilde{R}_{i,t+1}, \tilde{R}_{M,t+1}) = 122 \times \%^2,$$

and

$$Var_t(\tilde{R}_{M,t+1}) = 256\%^2.$$

Hence,

$$\beta_{i,t} = 122/256 = 0.4766.$$

2.6 Review of NPV analysis II

In the first part of the review we saw how to discount riskless cashflows.

Then we studied the basics of random variables, to understand the concept of risk better.

We are now ready to review the discounting of risky cashflows.

2.6.1 Discounting risky cashflows

We saw how the CAPM is used to related the excess expected return on one stock to the excess expected return on the market, via the systematic risk or beta of the stock.

$\mathrm{E}(\tilde{R}_i) - r = \beta_i(\mathrm{E}(\tilde{R}_M) - r)$, where
$\beta_i = \frac{\text{Cov}(\tilde{R}_i, \tilde{R}_M)}{\text{Var}(\tilde{R}_M)}$.

Suppose a publishing house, McGraw Hill wants to buy the software house Supersoft.

Supersoft has a an estimated beta of 1.4, the market risk premium is estimated to be 8.6% and the riskless rate is 5%. Assume a dividend growth rate of g.

How would you calculate the fair market value of Supersoft?

- You can use the CAPM to adjust the discount factor to value Supersoft.

- Then, the estimated discount rate for Supersoft is

  $5\% + 1.4 \times 8.6\% = 17.04\%$

- Estimated fair price per share:

  $\sum_{t=1}^{\infty} \frac{E(\tilde{D}_t)}{(1+17.04\%)^t} = \frac{(1+g)D_0}{17.04\%-g}$,

  using the formula for a perpetuity.

2.7 Valuation principles

2.7.1 Arbitrage pricing

Definition 2.2 (Arbitrage)
An arbitrage opportunity is one that guarantees the possibility of a positive profit without investing any of your own money and without taking any risk.

- An arbitrage opportunity is a free lunch.

- In developed financial markets, arbitrage opportunities are rare: financial instruments are priced in such a way that one cannot buy some and sell other securities so that:
  - the net investment is zero
  - the net risk is zero
  - the expected payoff is non-negative.

- Arbitrage says that two securities that have the same payoff must have the same price.

- Absence of arbitrage establishes relations $among$ security prices.

- We will use the concept of arbitrage to value a variety of derivative securities – you should be familiar with this from previous courses.

Note:

- Arbitrage pricing only prices one security off other securities – for example pricing a call option in terms of the underlying stock and bond.
- Arbitrage pricing does not determine all security prices – need to know the stock and bond prices before we can price the call option.

Key Assumptions of Arbitrage Pricing:

1. More is better than less (a weak assumption).

2. No frictions, such as

- trading costs
- short sales constraints.

2.7.2 Equilibrium pricing

- A second method for pricing securities is by imposing the condition that supply equals demand — market clearing in equilibrium.

- Market equilibrium determines security prices in terms of “fundamentals”
  - Expectation of future cash flows
  - Risk in future cash flows
  - Investors’ preferences toward risk
  - ...

Example 2.11 (Equilibrium pricing)
The Capital Asset Pricing Model (CAPM) is one framework used for equilibrium pricing.

Note:

- Equilibrium pricing allows us to price securities based on fundamentals.
- Equilibrium pricing determines <u>all</u> security prices.
- Equilibrium pricing is key to understanding economic forces behind security prices (e.g. expectations, risk, preferences).

2.8 Linear regression analysis

Relation between two random variables $\tilde{R}$ and $\tilde{R}_M$:

$$\tilde{R} = \alpha + \beta_{R,R_M}\tilde{R}_M + \tilde{e}$$

where

$$\boxed{\begin{aligned} \beta_{R,R_M} &= \frac{\text{Cov}[\tilde{R}, \tilde{R}_M]}{\text{Var}[\tilde{R}_M]} = \frac{\sigma_{R,R_M}}{\sigma^2_{R_M}} \\ \alpha &= \bar{R} - \beta_{R,R_M}\bar{R}_M; \quad \text{Cov}[\tilde{R}_M, \tilde{e}] = 0. \end{aligned}}$$

- β_{R,R_M} gives the expected deviation of $\tilde{R}$ from $\bar{R}$ for a given deviation of $\tilde{R}_M$ from $\bar{R}_M$; it is a measure of covariation between R_M and R
- $\tilde{e}$ has zero mean: $\mathrm{E}[\tilde{e}] = 0$

- $\tilde{e}$ represents the part of R that is uncorrelated with R_M:

$$\text{Cov}[\tilde{R}_M, \tilde{e}] = 0.$$

Furthermore:

$$\sigma_R^2 = \text{Var}[\tilde{R}] \ = \ \text{Var}[\alpha + \beta_{R,R_M}\tilde{R}_M + \tilde{e}] = \beta_{R,R_M}^2 \sigma_{R_M}^2 + \sigma_\epsilon^2$$

$$\text{Total Variance} \ = \ \text{Explained Variance} + \text{Unexplained Variance}.$$

- Explained variance: $\beta_{R,R_M}^2 \sigma_{R_M}^2$

- Unexplained variance: σ_ϵ^2.

- What fraction of the total variance of $\tilde{y}$ is explained by $\tilde{x}$?

$$R^2 = \frac{\text{Explained Variance}}{\text{Total Variance}} = \frac{\beta^2_{R,R_M}\sigma^2_{R_M}}{\sigma^2_R} = \frac{\beta^2_{R,R_M}\sigma^2_{R_M}}{\beta^2_{R,R_M}\sigma^2_{R_M} + \sigma^2_\epsilon}.$$

2.9 Long and short positions

Definition 2.3 (Long and short position)
A long position is one where you make money when the price increases; a short position is one where you make money when the price decreases.

Example 2.12 (Long and short positions)
Below, we list some positions and then state how a trader would describe them.

Actual position	Description
You will receive a Ferrari upon finishing this course	Long a Ferrari
You owe your friend a gold ring	Short gold
You have a GBP note in your pocket	Long spot GBP
You own a DEM deposit, maturing in 6 months	Long forward DEM
You promise to pay EUR 1 m in 6 months	Short forward EUR
You have sold a JPY bond (borrowed) in JPY	Short forward JPY
You have imported goods from Canada and have to pay for these goods in 3 months time	Short forward CAD

2.10 Appendix: Interest Rates, Returns, and Bond Yields

In this appendix, we summarize some basic but important information on interest rates. We use t to denote today, and T to denote the maturity date.

2.10.1 Interest Rates and Effective Returns

We have defined the (rate of) return as the percentage difference between the initial (time-t) value and the maturity (time-T) value of a nominally risk-free asset over a certain holding period. For instance, suppose you deposit \$ 100,000 for six months, and the deposit is worth \$ 105,000 at maturity. The six-month *effective return* is

$$\frac{105000 - 100000}{100000} = 5\%.$$

In reality, bankers never quote rates of returns; rather, they give you an interest rate. An $interest\ rate$ is an annualized return, that is, a return extrapolated to a 12-month horizon. In the text, we stress this by adding an explicit "$per\ annum$" (or $p.a.$) qualification whenever we mention an interest rate. However annualization can be done in many ways; and for any system, there is of course a corresponding way to de-annualize the interest rate into the return — the number you need.

Simple

Annualization can be *simple* (or linear): 5% for six months is just extrapolated linearly, to 10% per annum (p.a.). A simple interest rate is the standard method for term deposits and straight loans when the time to maturity is less than one year. Conversely, (unity plus) the return is computed from the quoted simple interest rate as

$$1 + r_{t,T} = 1 + [(T - t) \times (\text{simple interest rate})].$$

Example 2.13 (Simple interest)
Let $T - t = 1/2$ year, and the simple interest rate 10% p.a. Then

$$1 + r_{t,T} = 1 + [(1/2) \times 0.10] = 1.05.$$

Compound

Annualization can also be *compounded* (or exponential, with a hypothetical reinvestment of the interest). Using this convention, an increase from 100 to 105 in six months would lead to an extrapolated growth to $105 \times 1.05 = 110.25$ after another six months. Thus, 5% over six months corresponds to 10.25% p.a.. Conversely, (unity plus) the return is computed from the quoted compound interest rate as

$$1 + r_{t,T} = [1 + (\text{compound interest rate})]^{T-t}$$

Example 2.14 (Compound interest)
Let $T - t = 1/2$ year, and the compound interest rate 10.25% p.a. Then

$$1 + r_{t,T} = (1 + .1025)^{1/2} = 1.05.$$

Compound interest is the standard method for loans and investments (without interim interest payments) exceeding one year.

Banks can also compound the interest every quarter, or every month, or even every day. If the rate for a six-month investment is x per annum, compounded m times per year, the bank gives you x/m per subperiod of $1/m$ year.

$$1 + r_{t,T} = [1 + (\text{interest rate})/m]^{(T-t)\times m}.$$

Example 2.15 (Compound interest—frequency)
Let $T - t = 1/2$ year, and the quoted interest rate $=$ 9.878% p.a. to be compounded every quarter (i.e. $m = 4$). Then

$$1 + r_{t,T} = (1 + 0.09878/4)^{1/2\times4} = 1.05.$$

Continuous Compounding

In the theoretical literature, the frequency of compounding is often carried to the limit ("continuous compounding", $m \to \infty$). From your basic math course, you may remember that

$$\lim_{m\to\infty} [1 + (\text{interest rate})/m]^m = \exp(\text{interest rate})$$

where $\exp = e = 2.7182818$ is the base of natural (Naperian) logarithm. Conversely, (unity plus) the return is computed from the quoted interest rate as

$$1 + r_{t,T} = \exp[(\text{interest rate}) \times (T - t)].$$

Example 2.16 (Continuous compounding)
Let $T - t = 1/2$ year, and assume the continuously compounded interest rate equals 9.75803%. Then

$$1 + r_t = \exp(0.0975803 \times 1/2) = 1.05.$$

Banker's Discount

Banker's discount is yet another way of annualizing a return. This is often used when the present value is to be computed for T-bills, promissory notes, etc.—instruments where the time-T value (or face value) is known.

Suppose the time-T value is 100, the time to maturity $(T - t) = 0.5$ years, and the p.a. discount rate is 14%. The present value will then be computed as $100 \times (1 - 1/2 \times .14) = 93$. Or the discount, $100 - 93 = 7$ is just the face value (100), multiplied by $1/2 \times$ (banker's discount rate) $= 7\%$. Conversely, (unity plus) the return is found from the quoted banker's discount rate as

$$1 + r_{t,T} = \frac{(\text{face value})}{(\text{face value}) \times [1 - (T - t) \times (\text{discount rate})]}$$

$$= \frac{1}{1 - (T - t) \times (\text{discount rate})}.$$

Example 2.17 (Banker's discount)

Let $T - t = 1/2$ and the p.a. banker's discount rate $= 9.5238\%$. Then

$$1 + r_{t,T} = \frac{1}{1 - 1/2 \times 0.095238} = 1.05.$$

- To summarize,
  - There are many ways in which a bank can tell its customer that the effective return is, for instance, 5%.
  - Yet the effective return is all that matters.
  - Thus, interest rates or discount rates have to be de-annualized before you can effectively use them.

2.10.2 Common Pitfalls

Let us conclude with a discussion of the most frequently made mistakes.

1. The most common pitfall is that one forgets to de-annualize the return.

Thus, the first task always is to convert the bank's quoted interest rate into the effective return over the period $T - t$.

Example 2.18 (Different compounding frequencies)
Let $T - t = 0.5$ years. What are the effective rates of return when a banker quotes you a 12% per annum (p.a.) rate, to be understood as (i) simple interest; (ii) standard compound interest; (iii) interest compounded quarterly; (iv) interest compounded monthly; (v) interest compounded daily; (vi) interest compounded continuously; and vii) a banker's discount rate?

Bank's quote	$1 + R$	
12%, simple interest	$1 + 0.5 \times 0.12$	$= 1.06$
12%, compounded half-yearly	$(1 + 0.12)^{0.5}$	$= 1.05830$
12%, compounded quarterly	$(1 + 0.12/4)^2$	$= 1.06090$
12%, compounded monthly	$(1 + 0.12/12)^6$	$= 1.06152$
12%, compounded daily	$(1 + 0.12/360)^{180}$	$= 1.06183$
12%, continuously compounded	$\exp(0.12 \times 0.5)$	$= 1.06184$
12%, banker's discount	$\frac{1}{1-.5\times 0.12}$	$= 1.06383$

2. There is an interest rate (or a discount rate) for every maturity T.

For instance, if you make a 12-month deposit, the p.a. rate offered is likely to differ from the p.a. rate on a six-month deposit. People sometimes forget this, because basic finance courses occasionally assume, for expository purposes, that the p.a. compound interest rate is the same for all maturities. Thus, the second pitfall to be avoided is using a rate for the wrong maturity.

3. The third possible pitfall is to confuse an interest rate with an internal rate of return on a complex investment.

Recall that the return is the simple percentage difference between the maturity value and the initial value. This implicitly assumes that there is but one future cash flow. Yet many types of investments or loans carry many future cash flows. We shall discuss interest rates on multiple-payment instruments later in the course. At this stage, you should just remember that the interest rate on, say, a five-year loan with annual interest payments should not be confused with the interest rate on a five-year zero-coupon instrument.